LEVEL II

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(6) MAXIMUM POWER OUTPUT OF FIELD-LIMITED TRANSDUCER ELEMENTS.

by

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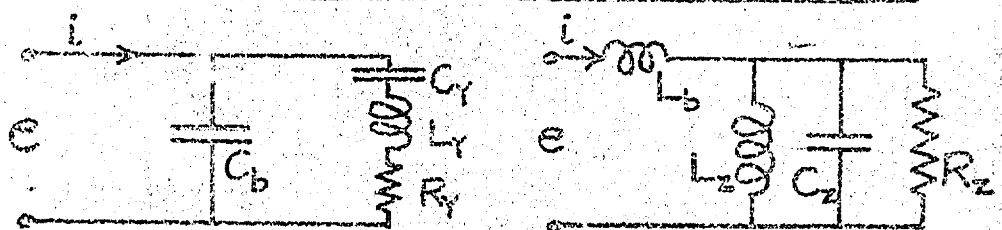
The following notes are published to illustrate methods currently in use for estimating the power output of transducer elements when the power is limited by the practically achievable electric or magnetic alternating field intensity. Where no units are given, rationalized mks units are assumed.

Definitions and Assumptions

(9) Technical memo.

This analysis is based on the use of the equivalent circuits shown below. Electromagnetic dissipation is neglected. The circuits describe only a single mode of vibration of the transducer element.

Electric-Field Transducer | Magnetic-Field Transducer



C_b = blocked capacity | L_b = blocked inductance

C_Y = motional capacity | L_z = motional inductance

L_Y = " inductance | C_z = " capacity

R_Y = " resistance | R_z = " resistance

Effective electromechanical coupling coefficient - k

$$k^2 = \frac{C_Y}{C_b + C_Y} \qquad k^2 = \frac{L_z}{L_b + L_z}$$

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Resonant frequency - $f_r = \omega_r/2\pi$

$$\omega_r^2 = \frac{1}{L_Y C_Y} \qquad\qquad \omega_r^2 = \frac{1}{L_Z C_Z}$$

Mechanical storage factor - Q_M

$$Q_M = \frac{1}{\omega_r C_Y R_Y} \qquad\qquad Q_M = \frac{R_Z}{\omega_r L_Z}$$

Electromagnetic stored energy (under quasi-static conditions) - U_e

$$U_e = \tfrac{1}{2} e^2 (C_b + C_Y) \qquad\qquad U_e = \tfrac{1}{2} i^2 (L_b + L_Z)$$

U_e does not include the electromagnetic energy stored in the polarizing field for those transducers which require polarization, or bias. e and i are the variational part of the voltage and current and do not include the polarizing voltage or current.

Mechanical Power at Resonance

The relation between the mechanical power at resonance and the input voltage or current can be given in terms of the parameters defined above as follows:

Electric-Field Transducer	Magnetic Field-Transducer
$P_M = \frac{1}{2} e^2/R_Y$	$P_M = \frac{1}{2} i^2 R_Z$
$= \frac{1}{2} e^2 (C_b + C_Y)\omega_r k^2 Q_M$	$= \frac{1}{2} i^2 (L_b + L_Z)\omega_r k^2 Q_M$

For both types of transducers:

$$P_M = \omega_r k^2 Q_M U_e$$

Maximum Power for Field-Limited Transducer

The equations developed above are based on linear transducer theory. They will now be used to calculate power for the large-signal case, where non-linearity is unquestionably present. This is a highly approximate procedure, but the results serve as a useful guide for most transducer designs.

For pulse applications, where heating is not a problem, the maximum driving field that may practically be employed is determined by such limits as breakdown of the dielectric or magnetizing coil, depolarization of the material for transducers operating at remanence, distortion tolerable in the input or output waveforms, and the tolerable loss of efficiency. Other limits such as elastic failure or cavitation at the radiating face are not considered in the present

analysis. The maximum driving field that may be used, based on these considerations, is designated E_{max} for electric field transducers and B_{max} for magnetic field transducers, and the corresponding electromagnetic stored energy is $(U_e)_{max}$.

$$(U_e)_{max} = \frac{1}{2}\epsilon^F E_{max}^2 V \qquad \text{electric-field transducers}$$

$$= \frac{1}{2}(1/\mu^F) B_{max}^2 V \qquad \text{magnetic-field "}$$

where

ϵ^F = free* permittivity

μ^F = free* permeability

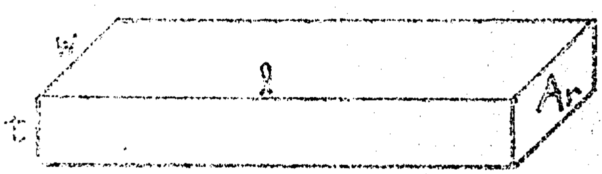
V = volume of active material.

Max. power at resonance: $P_M = \omega_r k^2 Q_M (U_e)_{max}$

*free of external constraints but with an internal stress distribution characteristic of the mode.

Longitudinally-Vibrating Half-Wavelength Bar

The case considered is the resonant bar radiating into water from one end only, but contained in an array of closely packed bars so that its radiating impedance Z_r is $\rho_w c_w A_r$.



ρ_w = density of water
ρ = " " bar
c_w = sound velocity of water
c = " " bar
A_r = radiating area of bar

$$Q_M = \frac{\pi}{2}\frac{\rho c}{\rho_w c_w} \qquad \text{for no internal dissipation.}$$

$$l = \frac{c}{2f_r} = \frac{\pi c}{\omega_r} \qquad V = l A_r = \frac{\pi c A_r}{\omega_r}$$

Relation between effective k and material k (k_m):

$$\frac{k^2}{1-k^2} = \frac{8}{\pi^2}\frac{k_m^2}{1-k_m^2}$$

When the field is in the direction of ℓ : $k_m = k_{33}$

" " " " " " " " w or t : $k_m = k_{31}$

For magnetostrictive bars, a low-reluctance magnetic return path must be provided in order to achieve the effective k given above.

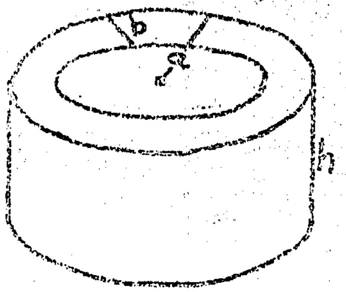
If no mechanical power is dissipated within the transducer, the surface intensity of radiation will be: $I_s = P_M/A_r$

$$I_s = \frac{\pi^2 \rho c^2 k^2 \epsilon^F E^2_{max}}{4 \rho_w c_w} \qquad \text{electrostrictive bar}$$

$$= \frac{\pi^2 \rho c^2 k^2 B^2_{max}}{4 \rho_w c_w \mu^F} \qquad \text{magnetostrictive bar}$$

Radially-Resonant, Short, Thin Ring

The case considered is the ring which is sufficiently short and thin so that it vibrates with uniform radial velocity, yet has sufficient outside area, in contact with water, that its radiation impedance Z_r is $\rho_w c_w A_r$.



$$Q_M = \frac{b}{a} \frac{\rho c}{\rho_w c_w} \qquad \text{for no internal dissipation.}$$

$$\omega_r = \frac{c}{a} \qquad V = bA_r \qquad A_r = 2\pi a h$$

The effective k equals the material k.
    for circumferential field: $k = k_{33}$
    for radial or axial field: $k = k_{31}$ (To operate magnetostrictive rings in this manner, a low-reluctance magnetic return path would have to be provided.)

Assuming no mechanical power dissipation:

$$I_s = \frac{P_M}{A_r} = \frac{\rho c^2 k^2 \epsilon^F E^2_{max}}{2 \rho_w c_w} \left(\frac{b}{a}\right)^2 \qquad \text{electrostrictive ring}$$

$$= \frac{\rho c^2 k^2 B^2_{max}}{2 \rho_w c_w \mu^F} \left(\frac{b}{a}\right)^2 \qquad \text{magnetostrictive ring}$$

Comparison of Ring and Half-Wavelength Bar

$$\frac{(I_s)_{bar}}{(I_s)_{ring}} = \frac{\pi^2}{2}\left(\frac{a}{b}\right)^2$$ for same material, same k, same driving field

The intensity for the bar is typically two orders of magnitude greater than that for the ring. This is accounted for by two factors: (a) the Q_M of the bar is much greater than the Q_M of the ring; (b) the volume of active material per unit area of radiating surface is much greater for the bar than for the ring.

Mechanical Power Derived from Maximum Generated Stress

A different approach, which is often used, is to start with a calculation of the maximum stress which can be generated piezoelectrically or magnetostrictively (without excessive non-linearity). Normally the same assumptions are made as in the preceeding general method, and identical results are produced. For example, a piezoelectric ring with radial field will generate a circumferential stress when the ring is blocked radially, which has the following value:

$$T_{\theta b} = \frac{d_{31}}{s_{11}^E} E_{max}$$

In transforming the circumferential stress to radial stress a step-down ratio (b/a) is effective; so the stress acting on the radial clamp is:

$$T_{rb} = \left(\frac{b}{a}\right) T_{\theta b} = \left(\frac{b}{a}\right)\frac{d_{31}}{s_{11}^E} E_{max}$$

At resonance this stress acts against the specific acoustic impedance of the water; thus:

$$P_s = -T_{rb} \qquad I_s = \frac{1}{2}\frac{P_s^2}{\rho_w c_w} = \frac{1}{2}\frac{d_{31}^2 E_{max}^2}{(s_{11}^E)^2 \rho_w c_w}\left(\frac{b}{a}\right)^2$$

Since $d_{31}^2 / s_{11}^E = k_{31}^2 \epsilon_{33}^T$ and $1/s_{11}^E = \rho c^2$, this result is the same as that found by the previous approach.

Numerical Examples

$BaTiO_3$ + 5% $CaTiO_3$ Ceramic

If heating is avoided, a maximum driving field of 2500 volts/cm r.m.s. is generally considered practical. The remanent polarisation is weakened if much higher fields are used. Other properties:

$$\epsilon^T = 1200 \times 8.85 \times 10^{-12} \times \left(\frac{1-k_{33}^2}{1-k^2}\right)$$

$k_{33} = .45$ $\qquad d_{33} = 140 \times 10^{-12}$

$\rho = 5500$ $\qquad c = 4500$ $\qquad s_{33}^E = 8.98 \times 10^{-12}$

Max. piezoelectric stress: $\quad T_b = \dfrac{d_{33}}{s_{33}^E} E = 15.6E$

$T_b = 5.5 \times 10^6$ newt/m^2 peak $= 730$ p.s.i. peak

Max. electromagnetic stored energy density:

$$(U_e)_{max}/V = \tfrac{1}{2}\epsilon^T E_{max}^2 = 640 \text{ joules/m}^3$$

Nickel, Soft-Annealed, with d.c. Polarization

Perhaps a peak driving flux density of 2000 gauss may be used before distortion and hysteresis loss become prohibitive, though definitive data on this point are still lacking. The average incremental permeability effective for this driving field can best be estimated by inspection of the major hysteresis loop. In this way it is seen that a change in magnetizing force of about 50 oersteds is needed to produce the total flux density change of 4000 gauss.

in e.m. units $\quad \mu_\Delta = \dfrac{2B_{max}}{\Delta H_+ + \Delta H_-} = \dfrac{4000}{50} = 80$

$\mu_{33}^T = 80 \times 4\pi \times 10^{-7} = 1.01 \times 10^{-4}$

$k_{33} = .30$ $\qquad g_{33} = \left(\dfrac{\partial S_3}{\partial B_3}\right)_{T=0} = 6.87 \times 10^{-5}$

$\rho = 8800$ $\qquad c = 4860$ $\qquad s_{33}^B = 4.81 \times 10^{-12}$

Max. magnetostrictive stress: $\quad T_b = \dfrac{g_{33}}{s_{33}^B} B = 1.43 \times 10^7 B$

$T_b = 2.86 \times 10^6$ newt/m^2 peak $= 415$ p.s.i. peak

Max. electromagnetic stored energy density:

$$(U_e)_{max}/V = \tfrac{1}{2}(1/\mu^T) B_{max}^2 = 198 \text{ joules/m}^3$$

Calculated Maximum Surface Intensity

	I_s
barium titanate $\lambda/2$ bar, k = .412	4000 Watts/cm^2
barium titanate striped ring, a/b = 5, k = .35	22 " "
nickel $\lambda/2$ bar, k = .272	1000 " "
nickel ring, a/b = 5, k = .30	8.2 " "

Comments on Results

The intensities calculated in these examples seem unrealistically high; so the various assumptions must be examined. In the case of the barium titanate bar, the assumption of field-limiting is violated, for it is found that the maximum stress exceeds the strength of the ceramic by at least a factor of two. The intensity for this element should then be reduced at least by a factor of four.

The general assumption of no mechanical power dissipation within the transducer is of course not valid; so the total power must be multiplied by the mechano-acoustical efficiency η_{ma} to obtain the radiated power. Actually, the intensity is reduced by the factor η_{ma}^2 from that calculated above, because not only the power but also the original Q_M must be multiplied by η_{ma} when mechanical losses are introduced. In practice, then, the intensity would be lower than that calculated above by a factor of 3 or 4 on account of mechanical dissipation.

When these corrections have been applied the resulting intensities should still be viewed as optimistic, because they are based on values of the transducer parameters measured at low level. The actual non-linearity will very likely act to reduce the power generated. Despite these reservations the methods described above are useful as a guide in preliminary transducer design, or in preliminary comparison of active materials.

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